

<p style="text-align: right;">A</p> <p>Representing, relating, and operating on whole numbers, initially with sets of objects</p>	<p style="text-align: right;">A</p> <p>Describing shapes and space</p>
<p style="text-align: right;">A</p> <p>Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20</p>	<p style="text-align: right;">A</p> <p>Developing understanding of whole number relationships and place value, including grouping in tens and ones</p>
<p style="text-align: right;">A</p> <p>Developing understanding of linear measurement and measuring lengths as iterating length units</p>	<p style="text-align: right;">A</p> <p>Reasoning about attributes of, and composing and decomposing geometric shapes</p>

<p style="text-align: right;">A</p> <p>Extending understanding of base-ten notation</p>	<p style="text-align: right;">A</p> <p>Building fluency with addition and subtraction</p>
<p style="text-align: right;">A</p> <p>Using standard units of measure</p>	<p style="text-align: right;">A</p> <p>Describing and analyzing shapes</p>
<p style="text-align: right;">A</p> <p>Developing understanding of multiplication and division and strategies for multiplication and division within 100</p>	<p style="text-align: right;">A</p> <p>Developing understanding of fractions, especially unit fractions (fractions with numerator 1)</p>

<p style="text-align: right;">A</p> <p>Developing understanding of the structure of rectangular arrays and of area</p>	<p style="text-align: right;">A</p> <p>Describing and analyzing two-dimensional shapes</p>
<p style="text-align: right;">A</p> <p>Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends</p>	<p style="text-align: right;">A</p> <p>Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers</p>
<p style="text-align: right;">A</p> <p>Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry</p>	<p style="text-align: right;">A</p> <p>Developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions)</p>

A

Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations

A

Developing understanding of volume

<p style="text-align: right;">A</p> <p>Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems</p>	<p style="text-align: right;">A</p> <p>Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers</p>
<p style="text-align: right;">A</p> <p>Writing, interpreting, and using expressions and equations</p>	<p style="text-align: right;">A</p> <p>Developing understanding of statistical thinking</p>
<p style="text-align: right;">A</p> <p>Developing understanding of and applying proportional relationships</p>	<p style="text-align: right;">A</p> <p>Developing understanding of operations with rational numbers and working with expressions and linear equations</p>

<p style="text-align: right;">A</p> <p>Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume</p>	<p style="text-align: right;">A</p> <p>Drawing inferences about populations based on samples</p>
<p style="text-align: right;">A</p> <p>Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations</p>	<p style="text-align: right;">A</p> <p>Grasping the concept of a function and using functions to describe quantitative relationships</p>
<p style="text-align: right;">A</p> <p>Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem</p>	

**A**

### Relationships between quantities and reasoning with equations

Students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

**A**

### Linear and exponential relationships

Students learn function notation and develop the concepts of domain and range.

They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

**A**

### Descriptive statistics

Students build upon prior experiences with data to develop more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

**A**

### Expressions and equations

Students build on their knowledge of exponents, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

**A**

### Quadratic functions and modeling

Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions - absolute value, step, and those that are piecewise-defined.

**A**

### Congruence, proof and constructions

Students build on prior knowledge to establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems – using a variety of formats – and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

**A**

### Similarity, proof, and trigonometry

Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem.

**A**

### Extending to three dimensions

Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

**A**

### Connecting algebra and geometry through coordinates

Students build on their work with the Pythagorean theorem in 8<sup>th</sup> grade to find distances, use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in the first course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

**A**

### Circles with and without coordinates

Students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done in the first course, to determine intersections between lines and circles or parabolas and between two circles.

**A**

### Applications of probability

Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

**A**

### Polynomial, rational, and radical relationships

Students develop the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

**A**

### Trigonometric functions

Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

**A**

### Modeling with functions

Students synthesize and generalize what they have learned about a variety of function families.

They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as “*the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions*” is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

**A**

### Inferences and conclusions from data

Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data - including sample surveys, experiments, and simulations - and the role that randomness and careful design play in the conclusions that can be drawn.

**A**

### Relationships between quantities

Students continue working with expressions and creating equations by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

**A**

### Linear and exponential relationships

Students learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

**A**

### Reasoning with equations

Students analyze and explain the process of solving an equation and justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and find and interpret their solutions.

**A**

### Descriptive statistics

Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

**A**

### Congruence, proof, and constructions

Students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons and apply reasoning to complete geometric constructions and explain why they work.

**A**

### Connecting algebra and geometry through coordinates

Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

**A**

### Extending the number system

Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. Students learn that when quadratic equations do not have real solutions, the number system must be extended so that solutions exist. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

**A**

### Quadratic functions and modeling

Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions – absolute value, step, and those that are piecewise-defined.

**A**

### Expressions and equations

Students focus on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

**A**

### Applications of probability

Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students make use of geometric probability models wherever possible. They use probability to make informed decisions.

**A**

### Similarity, right triangle trigonometry, and proof

Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. Students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

**A**

### Circles with and without coordinates

Students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

**A**

### Inferences and conclusions from data

Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data – including sample surveys, experiments, and simulations – and the role that randomness and careful design play in the conclusions that can be drawn.

**A**

### Polynomial, rational, and radical relationships

Students develop the structural similarities between the system of polynomials and the system of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. Students work with the fundamental theorem of algebra. Rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial.

A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

**A**

### Trigonometry and trigonometric functions

Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles opens up the idea of trigonometry applied beyond the right triangle – that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

**A**

### Mathematical modeling

Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as “the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions” is at the heart of this unit.

**In Kindergarten, instructional time should focus on:**

**In First Grade, instructional time should focus on:**

**In Second Grade, instructional time should focus on:**

**In Third Grade, instructional time should focus on:**

**In Fourth Grade, instructional time should focus on:**

**In Fifth Grade, instructional time should focus on:**

**B**

**In Kindergarten, instructional time should focus on:**

**B**

**In First Grade, instructional time should focus on:**

**In Second Grade, instructional time should focus on:**

**In Third Grade, instructional time should focus on:**

**B**

**In Fourth Grade, instructional time should focus on:**

**B**

**In Fifth Grade, instructional time should focus on:**

**In Sixth Grade, instructional time should focus on:**

**In Seventh Grade, instructional time should focus on:**

A

**In Eighth Grade, instructional time should focus on:**

B

**In Sixth Grade, instructional time should focus on:**

**B**

**In Seventh Grade, instructional time should focus on:**

**B**

**In Eighth Grade, instructional time should focus on:**

**In Mathematics I, instructional time should focus on:**

**In Mathematics II, instructional time should focus on:**

**In Mathematics III, instructional time should focus on:**

**In Algebra 1, instructional time should focus on:**

**In Geometry, instructional time should focus on:**

**In Algebra 2, instructional time should focus on:**

**In Mathematics I, instructional time should focus on:**

**In Mathematics II, instructional time should focus on:**

**In Mathematics III, instructional time should focus on:**

**In Algebra 1, instructional time should focus on:**

**In Geometry, instructional time should focus on:**

**In Algebra 2, instructional time should focus on:**